Abstract

Transformations in complex analysis can lead to geometric representations of vector fields on the complex plane. These interpretations can be quite helpful in the analysis of functions of complex variables. Many of the functions treated will be applications of harmonic functions. Specific topics addressed in the presentation will include two-dimensional fluid flows, Pólya-Latta vector fields, and the Joukowski airfoil. In addition to the main focus of hydrodynamics and fluid flow, interesting results in computer graphics and hydrodynamics will also be presented.

Introduction

Velocity Vector (vector field)

$$\vec{V}(x,y) = p(x,y) + iq(x,y)$$

Equation of Continuity

$$p_x(x,y) + q_y(x,y) = C$$

Complex Potential

$$f(x,y) = p(x,y) - iq(x,y)$$
$$F'(z) = f(z)$$
$$F(z) = \phi(x,y) + i\psi(x,y)$$
$$\overline{F'(z)} = \vec{V}(x,y)$$

Equipotentials and Streamlines describe the paths of fluid particles (κ_1, κ_2 vary within an interval)

Equipotentials (velocity potential) $\phi(x,y) = \kappa_1$
Streamlines (stream function) $\psi(x,y) = \kappa_2$

Invariance of Flow Theorem

Let

$$F_1(w) = \Phi(u,v) + i\Psi(u,v)$$

denote the complex potential for a fluid flow in a domain G in the w plane, where the velocity is

$$\vec{V}_1(u,v) = \overline{F_1'(w)}.$$

If the function

$$W = S(z) = u(x,y) + iv(x,y)$$

is a one-to-one conformal mapping from a domain D in the z plane onto F, then the composite function

$$F_2(z) = F_1(S(z)) = \Phi(u(x,y),v(x,y)) + i\Psi(u(x,y),v(x,y))$$

is the complex potential for a fluid flow in D, where the velocity is

$$\vec{V}_2(x,y) = \overline{F_2'(z)}.$$

Applications of Two-Dimensional Fluid Flow Mappings

- Joukowski airfoil
  - The image of a circle passing through $z=1$ and containing the point $z=-1$ is mapped onto a curve shaped like the cross section of an airplane wing
- Pólya-Latta vector fields
  - About polya-latta vector fields
  - Pólya plots can be created in *Mathematica* using **PlotPolyaField[f,{x,min,max},{y,min,max}]**