A COMPARISON OF A TEAM TEACHING APPROACH
TO A TRADITIONAL APPROACH IN
HIGH SCHOOL GEOMETRY

A Field Report
Presented to
The School of Graduate Studies
Drake University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Education

by
Robert J. Trusty
August 1971
A COMPARISON OF A TEAM TEACHING APPROACH
TO A TRADITIONAL APPROACH IN
HIGH SCHOOL GEOMETRY

by

Robert J. Trusty

Approved by Committee:

[Signatures]

Chairman

[Signature]

Joseph B. Holbert

Dean of the School of Graduate Studies

[3:1617]
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. THE PROBLEM AND DEFINITIONS OF TERMS.</td>
<td>1</td>
</tr>
<tr>
<td>THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>The Statement of the Problem.</td>
<td>1</td>
</tr>
<tr>
<td>Importance of the Study</td>
<td>2</td>
</tr>
<tr>
<td>Definition of Terms</td>
<td>3</td>
</tr>
<tr>
<td>Team Teaching</td>
<td>3</td>
</tr>
<tr>
<td>Learning Activity Package</td>
<td>3</td>
</tr>
<tr>
<td>Independent Learning</td>
<td>3</td>
</tr>
<tr>
<td>Small Group Learning</td>
<td>4</td>
</tr>
<tr>
<td>Large Group Instruction</td>
<td>4</td>
</tr>
<tr>
<td>The Control Group</td>
<td>4</td>
</tr>
<tr>
<td>The Experimental Group</td>
<td>4</td>
</tr>
<tr>
<td>II. PROCEDURE</td>
<td>5</td>
</tr>
<tr>
<td>The Organization</td>
<td>5</td>
</tr>
<tr>
<td>Selection of Experimental and Control Groups.</td>
<td>6</td>
</tr>
<tr>
<td>The Teaching Approaches</td>
<td>7</td>
</tr>
<tr>
<td>The Traditional Approach</td>
<td>8</td>
</tr>
<tr>
<td>The Team Approach</td>
<td>9</td>
</tr>
<tr>
<td>Selection and Evaluation of Test Questions</td>
<td>9</td>
</tr>
<tr>
<td>Validity of Tests</td>
<td>10</td>
</tr>
<tr>
<td>Reliability</td>
<td>11</td>
</tr>
<tr>
<td>III. REVIEW OF THE LITERATURE</td>
<td>13</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Advantages of Team Teaching</td>
<td>13</td>
</tr>
<tr>
<td>Disadvantages of Team Teaching</td>
<td>16</td>
</tr>
<tr>
<td>IV. RESULTS</td>
<td>19</td>
</tr>
<tr>
<td>Observations of the Program</td>
<td>19</td>
</tr>
<tr>
<td>Staff Use and Interaction</td>
<td>19</td>
</tr>
<tr>
<td>Independent Student Functioning</td>
<td>20</td>
</tr>
<tr>
<td>Small Group Instruction and Motivation</td>
<td>20</td>
</tr>
<tr>
<td>Large Group Instruction</td>
<td>21</td>
</tr>
<tr>
<td>Observations From the Comparison Phase</td>
<td>22</td>
</tr>
<tr>
<td>Reliability of Tests</td>
<td>22</td>
</tr>
<tr>
<td>Method of Comparison</td>
<td>22</td>
</tr>
<tr>
<td>V. SUMMARY AND CONCLUSIONS</td>
<td>24</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>26</td>
</tr>
<tr>
<td>APPENDIXES</td>
<td>29</td>
</tr>
<tr>
<td>A. SIMILARITY IN POLYGONS</td>
<td>30</td>
</tr>
<tr>
<td>B. SIMILARITY IN RIGHT TRIANGLES</td>
<td>34</td>
</tr>
<tr>
<td>C. TRIGONOMETRY OF THE TRIANGLE</td>
<td>37</td>
</tr>
<tr>
<td>D. COMPREHENSIVE EXAMINATION</td>
<td>40</td>
</tr>
</tbody>
</table>
CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS

The adoption of educational practices is dependent upon the willingness of classroom teachers to experiment. The improvement of techniques, which will allow for the best possible methods of instruction, is determined by the outcome of such experiments.

I. THE PROBLEM

The problem was to investigate the relative achievement of a class receiving instruction on a team teaching basis in high school geometry as compared to a similar group receiving instruction on a traditional basis.

The Statement of the Problem

The problem was threefold: (1) To observe a program in team teaching and to describe the program as it was carried out. (2) To compare the achievement of a group taught on the team teaching basis with a group taught in the traditional manner. (3) To test whether there is any difference in achievement between the two groups on a three section composite test and one comprehensive test.
Importance of the Study

The initial purpose of the project was to familiarize the faculty with team teaching. No member of the mathematics department faculty had experience in team teaching before the project. The opportunity to move into a new building with an instructional program designed by the faculty lead to the desire to gain some experience in different methods of instruction. Insight into the problems involved in team teaching gave the project considerable feasibility. The solutions to the problems involved gave the project importance.

The primary purpose of the project was to test whether there would be any difference in achievement between the students receiving team teaching instruction and a group of students that received instruction in the traditional manner. The knowledge that other methods of instruction are at least as good as the traditional method of instruction leads us to examine these methods for the attainment and development of other educational goals.

It was another important objective of the project to persuade the members of the faculty to contribute to the development of new methods of instruction. These methods would be developed to provide for goals not based on achievement of specific knowledge or ability to perform specific skills, but on the attainment of different objec-
tives. One such goal in adopting a team approach is to provide for more independent functioning by individual students.

II. DEFINITION OF TERMS

The following terms are defined in the way each was used throughout this project.

Team Teaching

Team teaching is the involvement of three classroom teachers in the planning for instruction and evaluation of student progress for a group of students.

Learning Activity Package

The Learning Activity Package (LAP) is a written guideline containing an exact statement of the behavioral objectives each student should achieve at the completion of the material, a schedule of lectures, suggested written assignments from the textbook, appropriate assignments from outside sources, and a self-administered test with complete solutions to all problems. The LAP was used only for the students in the team group.

Independent Learning

Independent learning is any activity a student may choose as a method of reaching the behavioral objectives. Independent learning includes use of written sources,
teachers, and other students. The emphasis is on student determination of the sources he may choose to attain the behavioral objectives.

**Small Group Learning**

Any activity involving more than one student and usually not more than fifteen students is classified as small group learning. This type of activity can be structured or unstructured by the supervisor.

**Large Group Instruction**

Traditional instruction is classified as instruction involving one teacher and one group of students meeting five days per week during the same period of time, with all the activities carried out under the direction of a single instructor.

**The Control Group**

Those students belonging to the traditionally taught group and who were selected for the comparison phase of the project.

**The Experimental Group**

Those students belonging to the team taught group and who were matched with the control group for the comparison phase of the project.
CHAPTER II

PROCEDURE

The procedure used in the experimental phase of the project will be described in this chapter. Many ideas were explored during the eighteen weeks prior to the experimental phase. These ideas were carried out in order to gain some insight into different ways of conducting the experimental phase.

I. THE ORGANIZATION

In May of the school year prior to the start of the project, the members of the mathematics department interested in the project met to begin the organizational structure. At this time, a schedule of topics to be used was discussed and a tentative plan for the beginning of school in the fall was made.

The students scheduled to take a general track course in geometry in the fall were assigned to one of the team members by the school registrar. The team teachers then combined three of the classes scheduled for the same period into the team group. One class scheduled to a member of the team during a different period was selected as the control group.

The need for a room in which to have large group presentations was satisfied by converting a basement storage
room to serve that purpose. Some changes in the room to meet fire regulations and the use of an overhead projector made the room usable for large group presentations.

For eighteen weeks, the team tried out different ideas to determine the procedure to be used during the comparison phase. The number of large group presentations and small group meetings were varied from one LAP to the next to help determine the best arrangement for our needs.

The ground work for the comparison phase was also completed during the eighteen week period. Team members were assigned to topics for large group presentations and selection of test questions was begun.

II. SELECTION OF EXPERIMENTAL AND CONTROL GROUPS

Selection of the experimental and the control groups was made from among those students who had chosen a general track geometry course and who had attended Dowling High School, Des Moines, Iowa, the previous school year. The high school is a non-public high school and the student body is composed of male students from the metropolitan area of Des Moines.

The team teaching group began with eighty-two students and the traditional group with twenty-five students. At the time of the comparison phase of the project, both the experimental group and the control group consisted of
twenty-four students. The control group lost one student whose schedule was changed. The experimental group was selected from the team teaching group to match the control group.

For experimental purposes, selected students in the team group were matched with students in the traditional (control) group. The matching was accomplished by equating standard scores on the fourth section of the Iowa Test of Educational Development. This section entitled, "Quantitative Thinking", was administered the previous school year. The group of students selected from the team teaching group will be referred to as the experimental group.¹

III. THE TEACHING APPROACHES

The teaching approaches used in the team and the control groups covered three units of high school geometry. Both approaches used Modern Geometry: Structure and Method as the textbook and as the primary source of written assignments.² The approaches were used for eighteen weeks prior to the experimental comparison phase of the project. This was done to reduce "Hawthorne" effects.

¹The University of Iowa, Iowa Test of Educational Development (tenth edition; Iowa City, Iowa: The University of Iowa, 1968).

The instructor involved in the traditional group was also a member of the team of teachers. To insure the best possible continuity of subject matter and testing sequence, the team of teachers was under the direction of the teacher who had the traditional group.

The major topics and subdivisions covered during the comparison phase were as follows:

1. Similarity in Polygons
   A. Properties of proportion.
   B. Similarity.
   C. Special segments in triangles.

2. Similarity in Right triangles
   A. Altitudes in right triangles.
   B. Pythagorean Theorem.
   C. 30-60-90 and 45-45-90 triangles.
   D. Projections.

3. Trigonometry of the Triangle
   A. Tangent ratio.
   B. Sine and cosine ratios.
   C. Applications of the trigonometric ratios.

Each major topic consisted of one LAP and took approximately two weeks to present.

The Traditional Approach

In the traditional approach the instruction was adjusted in content and time intervals to coincide with the team teaching approach.

The instructor in the traditional group presented the material, made assignments, carried out any discussion of
the assignments and upon completion of the subject matter tested for achievement. The traditional group instructor was involved in all planning stages of the tests used for both groups. All activities were carried out five days a week with forty-five minute sessions the general practice.

The Team Approach

In the team teaching approach, each teacher developed a learning activity package and presented lectures for the LAP. The LAP served as the guide for the activities of the instructors and students involved in the team approach.

The time intervals for each LAP consisted of two weeks, but were adjusted for interruptions due to holidays and assemblies. During the two week interval, a lecture was followed by two days for the development of the LAP objectives.

For the most part, the two days were not structured. Students were allowed to work by themselves or in small groups and teachers were available as resource personnel. These independent learning activities were also supplemented with structured discussion groups. However, use of the structured groups was an exception.

IV. SELECTION AND EVALUATION OF TEST QUESTIONS

Each teacher involved in the team teaching approach submitted twenty multiple-choice questions for each test
section. Questions submitted were then surveyed and duplicate questions were eliminated.

After the total number of questions was reduced to approximately forty, the questions were tested on a group of students not involved in the project.

Each question was evaluated as suggested by Joslin and others. Each question was eliminated if it failed to receive at least 20 per cent correct responses, if it received more than 90 per cent correct responses, or if more than 10 per cent failed to respond to the question.

Twenty-five of the questions which survived this first examination were then submitted to a second group of geometry students not involved in the project. The same criteria for elimination were again used on each question. The number of questions was further reduced to twenty.

Validity of Tests

The difficulty with tests devised on small scales is to show that the tests have validity. Validity has two factors which must be shown. Is the test relevant? How reliable are the results?

Each test and each section of each test was relevant to the extent that it had content validity. The use of each

---

question was determined by the extent to which it tested the behavioral objectives and by a consensus of the three teachers involved. The teachers, as professionals, determined that each question was justified for use on the tests.

After testing, it was discovered that some of the questions were ambiguous. The questions were devised for this project and in the context of the teaching situation did not appear to cause a problem.

Reliability

Reliability was determined by use of the Kuder-Richardson Formula 20. The Kuder-Richardson Formula 20 determines reliability by a single administration of a single form of a test by means of the formula:

\[ R = \frac{n}{n-1} \left( 1 - \frac{\sum P_i q_i}{s^2} \right) \]

\( s \) = standard deviation
\( P_i \) = proportion passing item
\( q_i \) = 1 - \( P_i \)
\( n \) = number of questions

To use the formula each question must have ample time to be answered. The number of questions for each test and test section was set at twenty to allow an average of more than two minutes for responding to each of the questions.

Test scores of all students were used to determine the reliability of each test. A three section composite test (one section for each LAP) and a comprehensive test
covering all three sections were used to compare the groups.
CHAPTER III

REVIEW OF THE LITERATURE

Why team teaching? Why new methods of instruction? The purpose of this chapter is to present the reasons for new methods, as well as main objections presented against some of the new methods.

I. ADVANTAGES OF TEAM TEACHING

The Easton Area High School Program used the team teaching approach as the instructional method to provide for five areas of concern. The program was developed to provide for the individual student, to provide each student with the opportunity to function independently, and improvement of student motivation. The Easton program attempted to provide for greater staff use and interaction. It was believed that these five concerns could be accomplished by team teaching.

Hanslovsky pointed out that in team teaching, "There is more unstructured time in which students may seek individual help from teachers and other special service per-

---

The use of time formally restricted to five classes a week could better be served by providing students with the opportunity for individual help.

One area of concern in mathematics teaching has been the availability of useful audio-visual materials. Most of the audio-visual materials are of little use in the specific presentations of content material.

The use of audio-visual aids and areas of large group presentations is enhanced because teachers have more time available in which to prepare for them. Since teachers are not required to present material to students five days a week and while other team members are involved with large group instruction, teachers are free to prepare for their next large group presentations.

"Flexibility of scheduling, especially the use of a large group presentation, releases time that may be used to explore new strategies of instruction." In the report from which the previous quotation was taken, the author confirmed that these advantages "were at least weakly observed."¹


Most grouping has been done on an ability basis. Once the grouping in the traditional class was accomplished, changing of such grouping was very difficult. The regrouping of some students has often been desirable because student abilities vary from topic to topic.

Employing small group instruction, team teachers should have a greater opportunity to regroup students more accurately and with greater ease. This was one of the objectives presented in the Joslin report to the Educational Research Association of New York State. The ability to regroup students under the team approach was not accomplished as fully desired by the team.

Another question to be answered was what effect such a project has on the students. How did the students accept the individual responsibility team teaching required? Could they adjust to the independent atmosphere? "Many feel that the chief yardstick for gauging the effects of team teaching is personality growth and adjustments." Personality growth has been the major justification of some team teaching programs.

---

1 Joslin and others, "Subjective Analysis of an Experiment Comparing a Team and a Conventional Approach to the High School Biology Course," *loc. cit.*

II. DISADVANTAGES OF TEAM TEACHING

Student adjustment has been of great importance to the success of most educational endeavors. "The judgment that a program is successful is based mainly on whether students "react with interest or enthusiasm" to the program." As long as schools judge programs on student acceptance, "... student achievement of operationally defined learning goals will have little influence on educational change."¹ Success or failure should be determined by evaluating attainment of the defined goals.

One of the major objections to most innovative changes has been the lack of instruments by which to measure the success of any program, "... there are many factors involved that will not yield to this (standarized achievement tests) testing procedure."²

Team teaching has been greatly dependent on large group presentations. The ability to use large group presentations to the greatest benefit has been an obstacle. "The basic things that we are aiming for in large group instruc-


² Kamb, op. cit., p. 53.
tions are far beyond just talking." Dr. J. Lloyd Trump continued, "the most important role . . . is to get students excited about learning." ¹ How many teachers are equipped to do this job successfully?

"Physical facilities account for some of the most persistently encountered disadvantages to team teaching." ² Old buildings, cost of remodeling, knowledge of what changes to make to do the best job--these are all barriers to doing the best job.

Some creative personnel object to working with a team of teachers because they claim that working with a team limits their freedom. Team teaching does require coordination of procedures and objectives. ³

Staff acceptance of change determines any progress in many innovations. School administrators find it very difficult to circumvent staff members who are not ready or who are unable to make the necessary adjustment to new programs.

¹Dr. J. Lloyd Trump, "The Large Group," (lecture given to the Oregon Program Team Teaching Workshop), Marshall High School, Marshall, Oregon, June 24-August 2, 1963, p. 2. (Mimeographed.)

²Dr. William Georgiades, "Staff Utilization Team Teaching," (lecture given to the Oregon Program Team Teaching Workshop), Marshall High School, Marshall, Oregon, June 26, 1963, p. 2. (Mimeographed.)

³Joslin and others, "Subjective Analysis of an Experiment Comparing a Team and a Conventional Approach to the High School Biology Course," loc. cit.
A teacher can give acceptance to working with a team, but unless he gives absolute effort to any team assignment, success will be limited.¹

¹Peterson, *op. cit.*, p. 177.
CHAPTER IV

RESULTS

This chapter describes the experiences of the team in this project. In spite of all the available material on team teaching, the benefits and difficulties in the development of a team teaching project can only be realized from experimentation. Each school situation is unique and the objectives of each school in establishing a team teaching project are somewhat different under each situation.

I. OBSERVATIONS OF THE PROGRAM

The following observations were made from the project as it was conducted. They were made from the areas of concern determined by the team members.

Staff Use and Interaction

The greatest advantage realized in this project was the mutual involvement of the staff in common problems. This was very satisfying to all staff members. Interaction in developing objectives served the purpose of unifying the team and this effect extended to the entire mathematics department.

The chief disadvantage the team encountered was the inability to meet on a regular basis. Provision for team planning and working together is extremely important for the
success of the team.

With the opportunity to meet together on a regular basis, at least once a week, many problems involving the interaction among team members could have been avoided. This opportunity to meet could help bring about the solution that best suits the entire team.

**Independent Student Functioning**

The students in the project were not accustomed to independent functioning. With more opportunities in other programs, ability in functioning independently could perhaps be increased.

The Learning Activity Package is designed to help the student in areas previously provided for by teachers in the traditional classroom. Time needs to be provided at the outset of a project for instruction in the use of the lap.

If provision for the individual is a chief factor in attempting innovative practices, helping the student learn on an independent basis must be provided. It will not just happen.

**Small Group Instruction and Motivation**

The attempt to allow students to choose the way in which each could best learn was not successful. Many of the students did not know how to help themselves learn. When this became apparent, the team then moved students into small group work directed by members of the team.
With small groups of ten students, the teacher could help motivate each student more effectively than he could in the traditional classroom and also establish greater rapport with each individual student in the small group.

**Large Group Instruction**

The purpose of the large group instructional period was to provide the available time for the small groups and for teachers to work on large group instructions. To make the best use of the large group, teachers and students need to adjust themselves mentally to the basic purpose, that is, to present basic material needed by a large majority of the group. The large group should probably not be used to review the course material unless the entire group needs the review.

Presentation of material in large groups should be different from presenting the same material as it is usually presented in the traditional classroom. Team members need to look for better ways of presenting material. This was one of the difficulties experienced by the team.

Time is available for each member of the team to prepare for his large group presentations. Team members have to look on the large group presentation as an important event. By approaching the large group with this attitude, each teacher could, as Dr. Trump expressed, "... get stu-
II. OBSERVATIONS FROM THE COMPARISON PHASE

The comparison phase of the project was designed to answer two questions. How does the achievement of the two groups compare on the two tests? Is there any difference in the achievement of the two groups?

Reliability of Tests

The reliability of the tests were determined by the Kuder-Richardson Formula 20. The reliability of the three section Composite Test was .731. The reliability of the Comprehensive Test was .702. The reliabilities were considered sufficient for the purpose intended. "A test with a reliability coefficient of only .70 would be satisfactory for comparing the average achievement of two groups or classes."²

Method of Comparison

The mean scores of the experimental group and the control group were compared for significance by the use of

¹Trump, loc. cit.

the "t" distribution for paired variates. Significance was determined at the .05 level. The entire group of students in the team teaching group were not used for comparison because they were not selected at random.

The "t" test was used for comparison in place of a parametric approach to help insure a reasonable equality in the comparison of achievement of the matched groups. The hypothesis being tested is whether there is a difference in the achievement of the two groups.

The mean scores of the two groups on the Composite tests were: 23.33 for the Experimental Group and 24.92 for the Control Group. Mean of the Control Group was larger than the mean of the Experimental Group but was not significant at the .05 level.

The mean scores of the two groups on the Comprehensive Test were: 9.96 for the Experimental Group and 11.04 for the Control Group. Again the mean of the Control Group was larger than the mean of the Experimental Group but was not significant at the .05 level.²


² Hodgeman, op. cit., p. 219.
The achievement of the experimental group as it compared to the achievement of the traditional group was not as large as expected by the team. The data appear to show that either teaching method is equally successful based on the achievement criteria.

Students did not appear to adjust to the freedom involved in the team approach. Having been closely directed by their previous teachers, the students were slow in adjusting to their new responsibilities.

Team members were not as effective as desired in the innovations involved. This is particularly true in the large group presentations. As has been previously stated, the large group presentations must be presented in a new and exciting manner.

In spite of the lack of complete success, there are sufficient reasons for conducting a new project in the team approach in the next school term. The reasons for conducting another team project are not based on the supposition that there will be better achievement in terms of knowledge but on the supposition that students can and should learn to function independently and for other objectives. The team members are planning to continue and to improve the tech-
niques necessary to make team teaching a completely successful experience.

More small projects of this nature are necessary if innovative practices are to become commonplace in most educational institutions. Each must be adapted to a particular set of conditions and students.

Development of Learning Activity Packages needs considerable study. Methods of instruction using the LAP should also be developed.

Techniques of large group presentations need improvement. The psychology of large group learning needs to be explored and results of such studies needs to be made available to the classroom teacher.

The development of instruments to measure the growth of the individual student involved in innovative practices is necessary for continual development of such practices.
BIBLIOGRAPHY

A. BOOKS


B. PERIODICALS


C. UNPUBLISHED WORKS

Geogiades, Dr. William. "Staff Utilization Team Teaching." Lecture given to the Oregon Program Team Teaching Workshop, Marshall, Oregon, June, 1963. (Mimeographed.)


Trump, Dr. J. Lloyd. "The Large Group." A lecture given to the Oregon Program Team Teaching Workshop, Marshall, Oregon, June, 1963. (Mimeographed.)

D. OTHER SOURCES


APPENDIXES
A. SIMILARITY IN POLYGONS

FOR EACH QUESTION, SELECT THE BEST ANSWER. USE ONLY ONE ANSWER PER QUESTION. THERE IS NO PENALTY FOR GUESSING, HOWEVER YOU CAN NOT BEAT THE ODDS.

ANSWER EACH QUESTION CAREFULLY.

FOR PROBLEMS 1-6; CONSIDER THE FOLLOWING PROPORTION:

\[
\frac{A}{C} = \frac{B}{D} \quad \text{AND} \quad C \neq D
\]

1. WHICH OF THE FOLLOWING IS NOT EQUIVALENT TO THE GIVEN PROPORTION?

- 1. \(A = B\)
- 2. \(A + B = C + D\)
- 3. \(A - B = C - D\)
- 4. \(\frac{B}{A} = \frac{D}{C}\)

2. WHICH OF THE FOLLOWING IS A CORRECT CONCLUSION FROM THE GIVEN PROPORTION?

- 1. \(A = B\)
- 2. \(A = D\)
- 3. \(A = C\)
- 4. \(C = B\)

3. WHICH OF THE FOLLOWING IS A CORRECT CONCLUSION FROM THE GIVEN PROPORTION?

- 1. \(\frac{A}{B} = \frac{C}{D} - \frac{A}{B}\)
- 2. \(\frac{A}{C} = \frac{C}{D} - \frac{A}{B}\)
- 3. \(\frac{A}{B} = \frac{C}{D}\)
- 4. \(\frac{A + C}{B} = \frac{A + C}{D}\)

4. IF \(A = 3\) AND \(B = 5\); THEN WHICH OF THE FOLLOWING IS A CORRECT CONCLUSION OF THE GIVEN PROPORTION?

- 1. \(C = 6\) AND \(D = 10\)
- 2. \(C = 3\)
- 3. BOTH ANSWERS 1 AND 2 ARE CORRECT
- 4. NONE OF THE ABOVE ANSWERS ARE CORRECT
5. IF $A = 2$, $B = 3$ AND $C = 4$; then which of the following is a correct conclusion of the given proportion?

1. $D = 9$  
2. $D = 5$  
3. $AD = 12$  
4. $AD = 6$

6. WHICH OF THE FOLLOWING IS A CORRECT CONCLUSION OF THE GIVEN PROPORTION?

1. $\frac{A - B}{B} = \frac{D - C}{D}$  
2. $AB = CD$  
3. $\frac{A}{B} = \frac{A + C}{B + D}$  
4. $\frac{A + B}{B} = \frac{C + D}{C}$

7. WHICH OF THE FOLLOWING STATEMENTS ARE TRUE?

1. $\frac{x^2}{3x} = \frac{3x^2}{1}$  
2. $\frac{3A^2B}{3AB^2} = \frac{B}{A}$  
3. $\frac{6x^2y}{2xy} = \frac{3x}{1}$  
4. $\frac{6(A^2B)^2}{3AB} = \frac{2c^2}{1}$

8. IF $5x = 6y$, THE RATIO OF $X$ TO $Y$ IS:  
1. $\frac{6}{5}$  
2. $\frac{5}{6}$  
3. $\frac{5x}{6y}$  
4. $\frac{6x}{5y}$

9. BY DEFINITION: TWO POLYGONS ARE SIMILAR IF:

1. TWO ANGLES OF ONE POLYGON EQUALS TWO CORRESPONDING ANGLES OF THE OTHER

2. EACH ANGLE OF ONE POLYGON EQUALS EACH CORRESPONDING ANGLE OF THE OTHER

3. CORRESPONDING SIDES ARE PROPORTIONAL

4. BOTH ANSWERS 2 AND 3 ARE NEEDED

FOR PROBLEMS 10-11; CONSIDER THE GIVEN FIGURE:

10. IF $BC = 18$, $YC = 10$ AND $AC = 9$; THEN $AX =$

1. 5  
2. 4  
3. 2  
4. 2 1/2

11. IF $BC = 7$, $YC = 4$ AND $AX = 8$; THEN $XC =$

1. 14  
2. 10  
3. 10 2/3  
4. 4 4/7
12. Which of the following represent the ratio of a side of a square to its area?

1. \( \frac{A}{2A} \)  
2. \( \frac{A}{4A} \)  
3. \( \frac{A}{A^2} \)  
4. \( \frac{4A}{A^2} \)

13. Which of the following are always similar?

1. Two triangles  
2. Two rectangles  
3. Two parallelograms  
4. None of the other answers are correct

14. Which of the following are not similar?

1. Congruent triangles  
2. Isosceles triangles  
3. Squares  
4. Circles

15. Similar triangles always have:

1. Equal corresponding angles  
2. Equal corresponding sides  
3. Both answers 1 and 2 are correct  
4. None of the above answers are correct

16. The altitudes of two similar triangles are:

1. Similar  
2. Equal  
3. Proportional to the perimeters  
4. None of these

17. What is the ratio of the perimeters of two squares with sides of 8 and 12?

1. 7:9  
2. 16:20  
3. 16:36  
4. 6:9

18. What is the ratio of the areas of two squares with sides of lengths 8 and 12?

1. 4:9  
2. 32:48  
3. 64:96  
4. 1:2
FOR PROBLEMS 19-20; CONSIDER THE GIVEN FIGURE:

GIVEN: \( \angle B = \angle D \)

PROVE: \( \frac{BA}{CD} = \frac{AE}{EC} \)

19. IN THE PROOF OF THIS PROBLEM, OF THESE FOUR STATEMENTS ARRANGED IN PROPER SEQUENCE; WHICH STATEMENT SHOULD APPEAR THIRD IN THE SEQUENCE?

1. \( \frac{BA}{CD} = \frac{AE}{EC} \)  
2. \( \angle AEB = \angle CED \)
3. TRIANGLE ABE IS SIMILAR TO TRIANGLE CDE
4. \( \angle B = \angle D \)

20. WHICH OF THE FOLLOWING REASONS JUSTIFIES THE STATEMENT, \( \angle AEB = \angle CED \)?

1. CORRESPONDING ANGLES OF SIMILAR TRIANGLES ARE EQUAL
2. IF TWO ANGLES OF A TRIANGLE EQUALS TWO ANGLES OF A SECOND TRIANGLE THE THIRD ANGLES ARE EQUAL
3. IF TWO ANGLES OF ONE TRIANGLE EQUALS TWO ANGLES OF A SECOND TRIANGLE THE TRIANGLES ARE SIMILAR
4. NONE OF THE ABOVE ANSWERS ARE CORRECT
B. SIMILARITY IN RIGHT TRIANGLES

For each question, select the best answer. Use only one answer per question. There is no penalty for guessing, however, you cannot beat the odds.

Answer each question carefully.

For problems 1-2; Right triangle ABC with right angle at C. Altitude segment CD

1. If \( 2b = c \), then:
   1. \( \angle CAD = 60^\circ \)
   2. \( \angle CBD = 60^\circ \)
   3. \( \angle DAC = 45^\circ \)
   4. \( \angle ACD = 60^\circ \)

2. If \( a = 3 \) and \( b = 4 \), then CD = ?
   1. \( 2 \frac{2}{5} \)
   2. \( 3 \frac{1}{5} \)
   3. \( 1 \frac{4}{5} \)
   4. \( 2 \frac{3}{5} \)

3. If two sides of a right triangle are 1 inch and 2 inches, which of the following could represent the third side?
   1. square root of 3
   2. square root of 5
   3. both answers 1 and 2
   4. none of these

4. The ratio of a side of a square to its diagonal is:
   1. \( \sqrt{2}/1 \)
   2. \( \sqrt{3}/3 \)
   3. \( \sqrt{2}/2 \)
   4. none of these

5. If a square is 6 inches on a side, the ratio of its area to its perimeter is?
   1. \( 4/1 \)
   2. \( 3/2 \)
   3. \( 2/1 \)
   4. none of these

6. If \( a \), \( b \) and \( c \) are the first three terms of a proportion in that order, the fourth proportional is?
   1. \( bc/a \)
   2. \( ac/b \)
   3. \( ab/c \)
   4. none of these

7. The mean proportional between \( a \) and \( b \) is:
   1. \( ab \)
   2. \( (ab)^2 \)
   3. \( \sqrt{ab} \)
   4. none of these
8. The projection of a segment onto a line could not be a:
   1. point 2. line 3. neither answers 1 and 2 are possible projections
   4. answers 1 and 2 are both possible projections

9. The projection of a rectangle onto a plane could not be a:
   1. segment 2. square 3. parallelogram
   4. trapezoid

10. If a segment 6 inches long makes an angle of 60 degrees with a plane, the length of its projection onto the plane is:
    1. 3 inches 2. \(3\sqrt{3}\) inches 3. \(3\sqrt{2}\) inches
    4. none of these

11. If one side of a square is 10 inches and if the side is projected onto the diagonal of the square, the length of the projection is:
    1. \(5\sqrt{2}\) 2. \(5\sqrt{3}\) 3. 5 4. none of these

12. Of the listed segments, the longest segment of a regular square pyramid is the:
    1. slant height 2. the altitude of a triangular face
    3. lateral edge 4. can not tell with the information given.

For problems 13-14: Segment CD is perpendicular to segment AB

Angle ACB is a right angle

13. Which of the following is a correct conclusion?
    1. triangle ADC is similar to triangle ACB
    2. triangle CDB is similar to triangle ACB
    3. triangle ADC is similar to triangle CDB
    4. all of these are correct conclusions
14. Which of the following is not a correct conclusion?
   1. \( \frac{CD}{AD} = \frac{DB}{CD} \)  
   2. \( \frac{AD}{AC} = \frac{AC}{AB} \)  
   3. \((CD)(AB) = (AC)(CB)\)  
   4. all of these are correct conclusions

15. An expression having a square root radical is in simplest form when:
   1. no integral radicand has a square factor other than 1  
   2. no fractions appear as radicand  
   3. no radicals are in a denominator  
   4. all of the above

16. In \( \sqrt[3]{A} \), "\( \sqrt{} \)" is called the:
   1. radical  
   2. radicand  
   3. index  
   4. none of these

17. Which of the following could not be the lengths of the sides of a right triangle?
   1. 6, 10, 8  
   2. 2, 2, \( 2\sqrt{2} \)  
   3. 2, 3, 5  
   4. all of the above are possible lengths

For problems 18-19; consider the following figure:

18. If \( CB = x \), then \( AC = ? \)
   1. \( 2x \)  
   2. \( x\sqrt{2} \)  
   3. \( x\sqrt{3} \)  
   4. none of these

19. If \( AB = x \), then \( AC = ? \)
   1. \( 2x \)  
   2. \( x\sqrt{2} \)  
   3. \( x\sqrt{3} \)  
   4. none of these

20. \( \sqrt[3]{\frac{2}{3}} = ? \)
   1. \( \sqrt[3]{2} \)  
   2. \( 3\sqrt[3]{3} \)  
   3. \( \sqrt[3]{6} \)  
   4. none of these
C. TRIGONOMETRY OF THE TRIANGLE

For each question, select the best answer. Use only one answer per question. There is no penalty for guessing, however, you can not beat the odds.

1. Sin A = ?
   1. cos(90° - A)  2. 1/cos A  3. both answers 1 and 2
   4. none of the other answers are correct

2. (tan A)(tan B) = ?
   1. 1  2. (tan A)(1/tan A)  3. both answers 1 and 2
   4. none of the other answers are correct

3. If a = 16 and c = 25, then angle A = ?
   1. 40°  2. 50°  3. 33°  4. 57°
   4. If b = 9 and a = 5, then angle B = ?
   1. 34°  2. 29°  3. 71°  4. 61°
   5. Cos 30° = ?
      1. 1/2  2. √3 /3  3. √3  4. √3 /2
   6. If angle A = 27° and c = 20, then b = ?
      1. 17.82  2. 9.08  3. 10.19
      4. None of these are approximately correct
   7. If angle A = 36° and b = 114, then a = ?
      1. 82.8  2. 92.2  3. 67.2  4. 156.9
8. If angle $B = 30^\circ$ and $a = 8$, then $c =$?
   1. $4\sqrt{3}$  
   2. $8\sqrt{3}$  
   3. $8\sqrt{3}/3$  
   4. $16\sqrt{3}/3$

9. If angle $B = 45^\circ$ and $c = 6$, then $a =$?
   1. $6\sqrt{3}$  
   2. 3  
   3. $3\sqrt{2}$  
   4. $2\sqrt{3}$

10. If $\sin A$ is less than $\sqrt{3}/2$, then:
    1. angle $A$ is less than $30^\circ$
    2. angle $A$ is greater than $30^\circ$
    3. angle $A$ is greater than $60^\circ$
    4. angle $A$ is less than $60^\circ$

11. If angle $A = 65^\circ$ and $a = 10$, then $b =$?
    1. 4.66  
    2. 21.4  
    3. 9.06  
    4. 4.20

12. Consider the following figure:
    \[
    \tan A = ?
    \]
    1. $3/4$  
    2. $4/3$  
    3. $4/5$  
    4. $3/5$

13. $\sin 30^\circ =$?
    1. $\cos 30^\circ$  
    2. $\tan 30^\circ$  
    3. $\tan 60^\circ$  
    4. $\cos 60^\circ$

For problems 14-15; consider the following figure:

14. If $AC = 10$ and angle $A = 40^\circ$, then $BC$ equals?
    1. 6.43  
    2. 1.5  
    3. 7.66  
    4. 5.77

15. If angle $A = 50^\circ$ and $AB = 10$, then $AC =$?
    1. 15.55  
    2. 6.43  
    3. 11.92  
    4. none of these are approximately correct
16. The vertex angle of an isosceles triangle is $80^\circ$ and the base is 20, what is the length of the altitude from the vertex angle?

1. 7.66  
2. 6.43  
3. 11.92  
4. none of these are approximately correct

17. In sighting the top of a lookout tower, the angle of elevation to the top of the tower is 53 degrees. If you are standing 210 ft. from the tower, how tall is the tower? Assume your eye level is 6 ft.

1. 285  
2. 132  
3. 279  
4. 134

18. In the following figure, what is the measure of angle CAB?

1. $69^\circ$  
2. $52^\circ$  
3. $62^\circ$  
4. $78^\circ$

19. $E = \frac{I \cos \Theta}{r^2}$ is a formula used in which of the following fields?

1. nuclear research  
2. lighting and illumination  
3. automotive engineering  
4. ballistics

20. The following figure shows three ships and the angles between sightings. If the distance between the ships at A and B is 47 miles, then the distance between ships A and C is:

1. 38 miles  
2. 35 miles  
3. 28 miles  
4. 43 miles
D. COMPREHENSIVE EXAMINATION

For each question, select the best answer. Use only one answer per question. There is no penalty for guessing, however, you can not beat the odds.

Answer each question carefully.

For problems 1-2; consider the following proportion: \( \frac{A}{B} = \frac{C}{D} \)

1. Which of the following is not equivalent to the given proportion?
   1. \( \frac{A}{C} = \frac{B}{D} \)  
   2. \( \frac{A - B}{B} = \frac{C - D}{D} \)  
   3. \( \frac{B}{A} = \frac{D}{C} \)  
   4. \( \frac{A - B}{B} = \frac{D - C}{D} \)

2. If \( A = 2, B = 3 \) and \( C = 4 \); then which of the following is a correct conclusion of the given proportion?
   1. \( AD = 6 \)  
   2. \( D = 9 \)  
   3. \( D = 5 \)  
   4. \( AD = 12 \)

3. In the given figure; If \( BC = 18 \), \( YC = 10 \) and \( AC = 9 \); then \( AX = ? \)
   1. 4  
   2. 2  
   3. 5  
   4. 2 1/2

4. Similar triangles always have:
   1. equal corresponding sides
   2. equal corresponding angles
   3. both answers 1 and 2 are correct
   4. none of the above answers are correct

5. What is the ratio of the perimeters of two squares with sides of 8 and 12?
   1. 7:9  
   2. 16:20  
   3. 16:36  
   4. 6:9
6. What is the ratio of the areas of two squares with sides of lengths 8 and 12?
   1. 4:9  2. 1:2  3. 64:96  4. 32:48

7. In right triangle, ABC, with right angle at C; what is the length of altitude CD, if BC = 3 and CA = 4?
   1. 1 4/5  2. 2 3/5  3. 2 2/5  4. 3 1/5

8. The mean proportional between a and b is:
   1. ab  2. (ab)^2  3. \sqrt{ab}  4. none of these

9. The projection of a rectangle onto a plane could not be a:
   1. segment  2. square  3. parallelogram  4. trapezoid

10. If a segment 6 inches long makes an angle of 60 degrees with a plane, the length of its projection onto the plane is:
    1. 3 inches  2. 3\sqrt{3} inches  3. 3\sqrt{2} inches  4. none of these

11. If one side of a square is 10 inches and if the side is projected onto the diagonal of the square, the length of the projection is:
    1. 5\sqrt{2}  2. 5\sqrt{3}  3. 5  4. none of these

12. Which of the following could not be the lengths of the sides of a right triangle?
    1. 6, 8, 10  2. 2, 3, 5  3. 2, 2, 2\sqrt{2}  4. all of the above are possible lengths

13. Consider the following figure:
    If CB = x, then AC = ?
    1. 2x  2. x\sqrt{2}  3. x\sqrt{3}  4. none of these
For problems 14-17; consider the following figure:

14. If \( a = 16 \) and \( c = 25 \), then angle \( A = ? \)
   1. 40 degrees  
   2. 50 degrees  
   3. 33 degrees  
   4. 57 degrees

15. \( \cos 30^\circ = ? \)
   1. \( \frac{1}{2} \)  
   2. \( \sqrt{3} \)  
   3. \( \frac{\sqrt{3}}{3} \)  
   4. \( \frac{\sqrt{3}}{2} \)

16. If angle \( A = 27 \) degrees and \( c = 20 \), then \( b = ? \)
   1. 17.82  
   2. 10.19  
   3. 9.08  
   4. none of these are approximately correct

17. If angle \( B = 45 \) degrees and \( c = 6 \), then \( a = ? \)
   1. \( 6\sqrt{3} \)  
   2. 3  
   3. \( 3\sqrt{2} \)  
   4. \( 2\sqrt{3} \)

18. The vertex angle of an isosceles triangle is 80 and the base is 20, what is the length of the altitude from the vertex angle?
   1. 7.66  
   2. 6.43  
   3. 11.92  
   4. none of these are approximately correct

19. In the following figure, what is the measure of angle \( CAB \)?
   1. 69°  
   2. 52°  
   3. 62°  
   4. 78°
20. The following figure shows three ships and the angles between sightings. If the distance between ships at A and B is 47 miles, then the distance between ships A and C is:

1. 28 miles  
2. 43 miles  
3. 38 miles  
4. 35 miles